Maths Calculation Guidance for Datchet St Mary’s C of E Primary Academy

January 2017

Datchet St Mary’s C of E Primary Academy Maths Calculation Policy

Introduction

At Datchet St Mary’s we believe that children should be introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children’s mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave Datchet St Mary’s is that they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a ‘feel’ for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall key number facts instantly - for example, all addition and subtraction facts for each number to at least 20 (Year 2), sums and differences of three digit numbers and ones, tens and hundreds (Year 3) and multiplication facts up to 12x12 (Year 4);

Written methods of calculation

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as 'standard' written methods - methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that we adopt greater consistency in our approach to calculation. The challenge is for our teachers to determine when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

Talking:

- There is evidence that peer interactions are a main facilitator factor for socio-cognitive development and their performance in maths tasks.
- A classroom culture of questioning in which pupils learn from shared discussions with teachers and peers.
- Teacher's role helps pupils to feel free and confident to comment and to suggest strategies even if there are errors in calculations. This allows for all pupils to reflect and suggest more appropriate methods.
- To encourage teachers to value all contributions and be willing to change their own minds in the light of what the pupil says.
- Tasks are planned so there are opportunities for pupils to communicate their evolving understanding.

We value the communication between teachers and pupils, pupils and their peers. When children feel confident to communicate their ideas and discuss their findings openly this improves their level of understanding. It has been proved that children will remember 70% of what they have been learning if they taken an active part in the lesson compared to a passive learner who will only retain 20% of what has been taught.

Vocabulary

Vocabulary builds as the children progress through the school and the vocabulary highlighted for each year group are the most frequently used words.

Choosing the appropriate strategy

Recording in mathematics, and in calculation in particular is an important tool both for furthering the understanding of ideas and for communicating those ideas to others. A useful written method is one that helps children carry out a calculation and can be understood by others. Written methods are complementary to mental methods and should not be seen as separate from them. The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. It is important children acquire secure mental methods of calculation and one efficient written method of calculation for addition, subtraction, multiplication and division which they know they can rely on when mental methods are not appropriate. As a long term aim children should be able to choose an efficient method; mental, written -that is appropriate to a given task (using calculators to check answers).

Objectives

The objectives in the revised Framework show the progression in children's use of written methods of calculation in the strands 'Using and applying mathematics' and 'Calculating'.

EYFS follow their own curriculum. Refer to the EYFS Curricuum for more information.

Using and applying mathematics	Calculating
Year 1 • Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change' • Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context	**Year 1** • Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number • Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number • Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences
Year 2 • Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence • Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem	**Year 2** • Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders • Use the symbols +, -, ×, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. … ÷ 2 = 6, 30 - … = 24)
Year 3 • Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations • Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or units of measure	**Year 3** • Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers • Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13 × 3, 50 ÷ 4); round remainders up or down, depending on the context • Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences

Using and applying mathematics	Calculating
Year 4 • Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculators to check work where appropriate • Represent a puzzle or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem	**Year 4** • Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p • Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15 × 9, 98 ÷ 6)
Year 5 • Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, using calculators to check work where appropriate. • Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem	**Year 5** • Use efficient written methods to add and subtract whole numbers and decimals with up to two places • Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 or 1000 • Refine and use efficient written methods to multiply and divide HT1s × 1s, T1s × T1s, 1s.t × 1s and HT1s ÷ 1s
Year 6 • Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, using calculators to check work where appropriate. • Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of c pens at 15 pence each is $15c$ pence)	**Year 6** • Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer

Written methods for addition

These phases show the building up to using an efficient written method for addition of whole numbers by the end of Year 4.

To add successfully, children need to be able to:

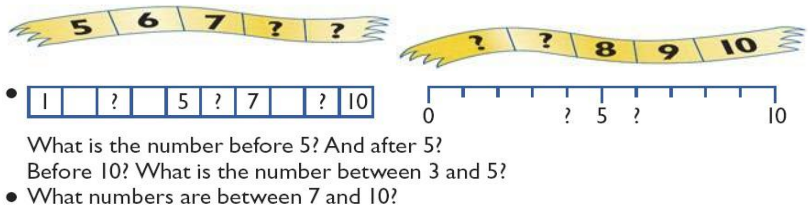
- recall all addition pairs to 9 + 9 and complements in 10 (making ten e.g. 4+6=10);
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)..

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Phase 1

Develop secure one-one correspondence and understanding of addition.

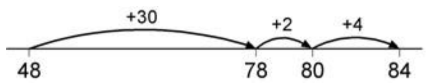
- **Count accurately 0-10**
- **Recognise and write numerals 1-10**



- Count and add together sets of real objects and pictures.

3+2 = 5

+

Phase 2: The number line and 100 square The number line helps children to move from using concrete objects. Children begin to split a number to add to the nearest multiple of 10 and then count on. The 100 square supports children's understanding when adding ten to any number the ones stay the same and the tens go up. Eventually this will be done mentally.	**Phase 2** To use a number line to add one or more numbers together. 8+1 = 9 +1 0 1 2 3 4 5 6 7 8 9 10 • To be able to add through 10. 8+5 = 13 +2 +3 0 1 2 3 4 5 6 7 8 9 10 11 12 13 *100 Square* To be able to add 10 to any number up to 100 using a 100 square by initially counting on 10. 9+10 = 10+9 = 19 To be able to add multiples of 10 to any number up to 100 using a 100 square. 34+40 = 74 **(100 square picture?)**	**Key vocabulary** Add, more, count on, plus, sum, total, altogether, partition, how many. Multiple of 10,number line, 100 square
Phase 3: The empty number line • The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps. • The empty number line helps to record the steps on the way to calculating the total.	**Phase 3** Steps in addition can be recorded on an empty number line. The steps often bridge through a multiple of 10. 8 + 7 = 15 To be able to add 2 two digit numbers on an empty number line.  48 + 36 = 84  or: 	***As above***

Phase 4: Partitioning • The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums. • Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.	**Phase 4** Record steps in addition using partitioning: 47 + 76 = 47 + 70 + 6 = 117 + 6 = 123 or 47 + 76 40 + 70 = 110 7 + 6 = 13 = 123 Partitioned numbers are then written under one another: 47 = 40 + 7 + 76 70 + 6 110 +13 =123	***As above*** column, addition, tens boundary **Teaching point** Ensure correct use of the = sign.
Phase 5: Expanded method in columns • Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums either the tens or the ones can be added first, and the total of the partial sums can be found by adding them in any order. As children gain confidence, ask them to start by adding the ones digits first always. • The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.	**Phase 5** To add two digit numbers in vertical layout Write the numbers in columns. Adding the tens first: 47 + 76 110 13 123 OR Adding the ones first: 47 + 76 13 110 123 Discuss how adding the ones first give the same answer as adding the tens first. Refine over time to adding the unit digits first consistently. Develop on to adding three digit numbers in vertical layout. Extend to use decimals to 2 decimal places. 24.68 +17.94 0.12 1.50 11.00 30.00 42.62	***As above*** Hundreds boundary, approximate
Phase 6: Column method • In this method, recording is reduced further. Carry digits are recorded in the columns above the numbers, using the words 'carry ten' or 'carry one hundred', not 'carry one'. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	**Phase 6 (Support method)** **H T1s** **1 1** **4 7** By leaving a space before the calculation **+ 7 6** allows the children to carry the number **1 2 3** in the correct column and clearly see this. Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.	***As above*** Carry,

Phase 6 contd **Phase 7: Column method expanded to decimals** In this phase the standard method is expanded to include the use of decimals.	789+642 **Becomes** 789 +642 _______ 1431 -------- 1 1 **Phase 7 (Support method) or with carried numbers underneath for** **1 1** 45.24 + 26.59 71.83	***As above*** ones boundary, tenths boundary, hundredths boundary.

Written methods for subtraction

These Phases show the building up to using an efficient method for subtraction of two-digit and three digit whole numbers by the end of Year 4.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 - 70) using the related subtraction fact,16 - 7, and their

  knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

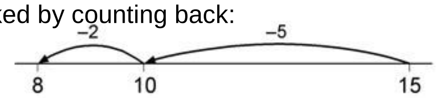
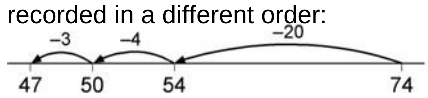
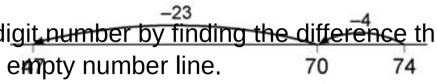
Phase 1

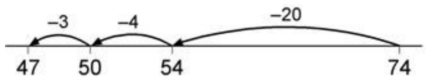
Develop secure one-one correspondence and understanding of subtraction.

- **Count accurately forwards and backwards from 0-10**
- **Recognise and write numerals 1-10**

- Count and subtract sets of real objects and pictures.

5-2 = 3

Phase 2 - number line and 100 square	**Phase 2** To be able to subtract/take away one less on a number line. 8-1 = 7 -1 0 1 2 3 4 5 6 7 8 9 10 To be able to subtract through 10 13-5 = 8 -2 -3 0 1 2 3 4 5 6 7 8 9 10 11 12 13 To be able to subtract 10 from any number up to 100 using a 100 square. 34-10=24 To be able to subtract multiples of 10 from any number up to 100 using a 100 square. 84-40=44 (100 square)	
Phase 3: Using the empty number line • The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 - 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. • The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. • With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 - 12, 86 - 77 or 43 - 28.	**Phase 3** Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. 15 - 7 = 8  74 - 27 = 47 worked by counting back:  The steps may be recorded in a different order:  or combined:  Subtract a 2 two digit number by finding the difference through counting on an empty number line. 45-23= +7 +10 +5 23 30 40 45 10+7+5=22	

Phase 4: Partitioning • Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 - 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.	**Phase 4** Subtraction can be recorded using partitioning: 74 - 27 = 74 - 20 - 7 = 54 - 7 = 47 This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line. 	subtraction, difference, minus, more than,
Stage 5: Expanded layout, leading to column method • Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens. • This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.	**Stage 5** Partitioned numbers are then written under one another: Example: 74 − 27 60 14 6 14 70 + 4 ~~70~~+ ~~4~~ 7 4 − 20 + 7 − 20 + 7 − 27 40 + 7 4 7 Example: 741 − 367 600 130 11 6 13 11 700 + 40 +1 ~~700~~+ ~~40~~ + ~~1~~ 7 4 1 − 300 + 60 + 7 − 300 + 60 + 7 − 367 300 + 70 + 4 374 You may find that the children are able to move straight to the compact method 87 – 52 = EMPHASISE START WITH THE ONES 87 -52 35 932 – 457 becomes 932 or 932 - 457 - 457	exchange

The expanded method for three-digit numbers

Example: 563 − 241, no adjustment or decomposition needed

Expanded method leading to

$$\begin{array}{r} 500 + 60 + 3 \\ \underline{-\ 200 + 40 + 1} \\ 300 + 20 + 2 \end{array} \qquad \begin{array}{r} 563 \\ \underline{-\ 241} \\ 322 \end{array}$$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying ‘sixty take away forty’, not ‘six take away four’.

Example: 563 − 271, adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} 500 + 60 + 3 \\ \underline{-\ 200 + 70 + 1} \\ \end{array} \quad \begin{array}{r} 400 + 160 + 3 \\ \underline{-\ 200 + 70 + 1} \\ 200 + 90 + 2 \end{array} \quad \begin{array}{r} \overset{400}{\cancel{500}} + \overset{160}{\cancel{60}} + 3 \\ \underline{-\ 200 + 70 + 1} \\ 200 + 90 + 2 \end{array} \quad \begin{array}{r} \overset{4}{\cancel{5}}\,\cancel{6}\,{}^{16}3 \\ \underline{-\ 271} \\ 292 \end{array}$$

Begin by reading aloud the number from which we are subtracting: ‘five hundred and sixty-three’. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes ‘160 minus 70’, an application of subtraction of multiples of ten.

Example: 563 − 278, adjustment from the hundreds to the tens and the tens to the ones

$$\begin{array}{r} 500 + 60 + 3 \\ \underline{-\ 200 + 70 + 8} \\ \end{array} \quad \begin{array}{r} 400 + 150 + 13 \\ \underline{-\ 200 + 70 + 8} \\ 200 + 80 + 5 \end{array} \quad \begin{array}{r} \overset{400}{\cancel{500}} + \overset{150}{\cancel{60}} + \overset{13}{\cancel{3}} \\ \underline{-\ 200 + 70 + 8} \\ 200 + 80 + 5 \end{array} \quad \begin{array}{r} \overset{4}{\cancel{5}}\,\overset{15}{\cancel{6}}\,\overset{13}{\cancel{3}} \\ \underline{-\ 278} \\ 285 \end{array}$$

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 − 278, dealing with zeros when adjusting

$$\begin{array}{r} 500 + 0 + 3 \\ \underline{-\ 200 + 70 + 8} \\ \end{array} \quad \begin{array}{r} 400 + 90 + 13 \\ \underline{-\ 200 + 70 + 8} \\ 200 + 20 + 5 \end{array} \quad \begin{array}{r} \overset{400}{\cancel{500}} + \overset{90}{\cancel{0}} + \overset{13}{\cancel{3}} \\ \underline{-\ 200 + 70 + 8} \\ 200 + 20 + 5 \end{array} \quad \begin{array}{r} \overset{4}{\cancel{5}}\,\overset{9}{\cancel{0}}\,\overset{13}{\cancel{3}} \\ \underline{-\ 278} \\ 225 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

Teaching point;

Ensure that the pupils are secure at the expanded stage before progressing to the standard decomposition method

Our children are encouraged to use decomposition when they have a secure understanding of place value in relation to subtraction.

$$\begin{array}{r} 473 \\ -\ \underline{389} \\ 84 \end{array}$$

Written methods for multiplication of whole numbers

These phases show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

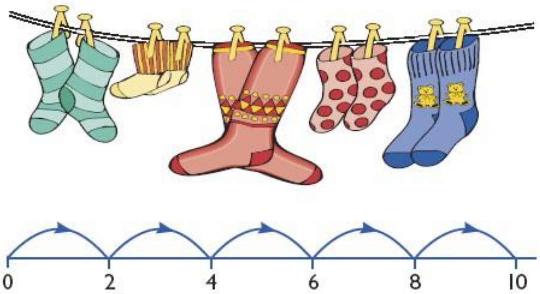
- recall all multiplication facts to 12 × 12;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Phase 1: Hands on experiences

Initially children put objects into groups/ sets.

Solve multiplication through repeated addition.



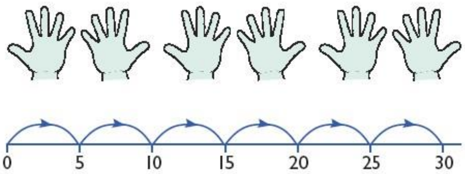
$2+2+2+2+2=10$

$2\times5=10$

2 multiplied by 5

5 pairs

5 hops of 2



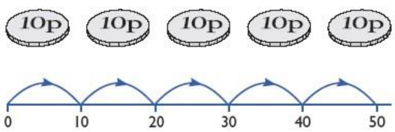
$5+5+5+5+5+5=30$

$5\times6=30$

5 multiplied by 6

6 groups of 5

6 hops of 5

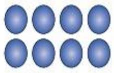


10p + 10p + 10p + 10p + 10p = 50p

10p × 5 = 50p

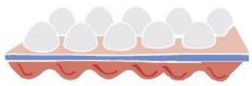
5 hops of 10

To use arrays to solve multiplication problems.



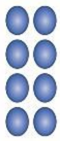
$4\times2=8$

$2\times4=8$



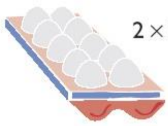
$5\times2=10$

$2\times5=10$



$2\times4=8$

$4\times2=8$

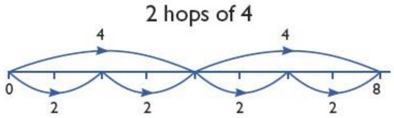


$2\times5=10$

$5\times2=10$

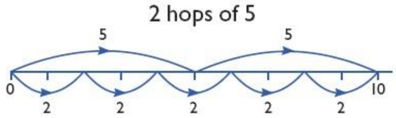
Children begin to identify patterns within multiplications (e.g. number always ends with a 0, 2, 4, 6 or 8 in the 2X table) and between multiplications (4X table is double the 2X table).

2 hops of 4



4 hops of 2

2 hops of 5



5 hops of 2

To multiply by 4

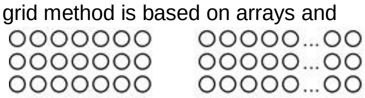
17x4=

17x2x2= 34x2 =68

To multiply by 5

14x5=

(14x10)/2= 140/2 =70

Phase 2: Mental multiplication using partitioning • Mental methods for multiplying T1s × 1s can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products.These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens. Note: These methods are based on the distributive law. Children should be introduce the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times- tables to work out multiples of 7: 7×3 = (5 + 2)×3 = (5×3)+(2×3) =15 + 6 = 21	Phase 2 Informal recording in Year 4 will look like this: 43 40 + 3 ×6 240 + 18 = 258 Also record mental multiplication using partitioning: 14×3 = (10 + 4)×3 = (10×3)+(4×3) = 30 +12 = 42 43×6 = (40 + 3)×6 = (40×6)+(3×6) = 240 +18 = 258	Key Vocabulary product multiplication, array, partition,
Phase 3: The grid method • As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps. • It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products. NOTE IF YOU PUT AN ADDITION SIGN BETWEEN THE TWO BOXES WITHIN THE GRID IT HELPS CHILDREN REMEMBER TO ADD THE TWO NUMBERS. This method may help children less secure, who are unable to move to the short form of multiplication.	**Phase 3** To know how the grid method is based on arrays and extend the use  of arrays to the grid method.3x12 = (3x10) + (3x2) = 36 X 10 2 3 30 6 30 6 36 X 10 2 3 30 6 30 6 + 36 Grid method To multiply 2 digit number by a 1 digit number using the grid method. 38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266 × 7 30 210 8 56 266	**As above** grid method, partition, multiplication, column. – –

<table>
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The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56.
The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.

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<table>
<tr><td></td><td>30 + 8</td></tr>
<tr><td>×</td><td>7</td></tr>
<tr><td></td><td>210</td></tr>
<tr><td></td><td>56</td></tr>
<tr><td></td><td>266</td></tr>
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<td>As above</td>
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Phase 4: Expanded short multiplication

The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is ‘thirty multiplied by seven’, not ‘three times seven’, although the relationship 3 × 7 should be stressed.
Most children should be able to use this expanded method for T1s × 1s by the end of Year 4.

</td>
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ENCOUNRAGE HABIT OF STARTING WITH ONES NUMBER, as this moves more naturally to the shortened form.
Phase 4
30 + 8   38
× 7   × 7
56   8 x7 =56   56
<u>210</u>   30×7 = 210   <u>210</u>
<u>266</u>   <u>266</u>
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Phase 5: Short multiplication

The recording is reduced further, with carry digits recorded below the line.
If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 4.

</td>
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Phase 5
38
× 7
<u>266</u>
5
The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.
</td>
<td>As above

Carry,</td>
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<td>Phase 6: Two-digit by two-digit products
• Extend to T1s × T1s, asking children to estimate first.
• Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
• As in the grid method for T1s × 1s in stage 4, the first column can become an extra top row as a stepping stone to the method below.</td>
<td>Phase 6
56 × 27 is approximately 60 × 30 = 1800.</td>
<td><table><tr><td>×</td><td>20</td><td>7</td><td></td></tr><tr><td>50</td><td>1000</td><td>350</td><td>1350</td></tr><tr><td>6</td><td>120</td><td>42</td><td>162</td></tr><tr><td></td><td></td><td></td><td>1512</td></tr><tr><td></td><td></td><td></td><td>1</td></tr></table></td>
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<tr>
<td>• Reduce the recording, showing the links to the grid method above.</td>
<td>56 × 27 is approximately 60 × 30 = 1800.
56
× <u>27</u>
1000   50×20 = 1000
120   6×20 = 120
350   50×7 = 350
<u>42</u>   6×7 = 42
<u>1512</u>
1</td>
<td><table><tr><td></td><td>~~50~~</td><td>~~6~~</td><td></td></tr><tr><td>×</td><td>20</td><td>7</td><td></td></tr><tr><td></td><td>1000</td><td>350</td><td>1350</td></tr><tr><td></td><td>120</td><td>42</td><td>162</td></tr><tr><td></td><td></td><td></td><td>1512</td></tr><tr><td></td><td></td><td></td><td>1</td></tr></table></td>
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<tr>
<td>• Reduce the recording further.
• The carry digits in the partial products of 56 × 20 = 1120 and 56 × 7 = 392 are usually carried mentally.
• The aim is for most children to use this long multiplication method for T1s × T1s by the end of Year 5.</td>
<td>56 × 27 is approximately 60 × 30 = 1800.
56
× <u>27</u>
1120   56×20
<u>392</u>   56× 7
<u>1512</u>
1</td>
<td></td>
</tr>
<tr>
<td>Phase 6: Three-digit by two-digit products
• Extend to HT1s × T1s asking children to estimate first. Start with the grid method.
• It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.
• Children may need to do a separate recording when adding two sets of numbers together such as
1600
+ <u>720</u></td>
<td>Phase 6
286 × 29 is approximately 300 × 30 = 9000.</td>
<td></td>
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</table>

• Reduce the recording, showing the links to the grid method above. • This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.	286 × 29 4000 200×20 = 4000 1600 80×20 = 1600 120 6×20 = 120 1800 200× 9 = 1800 720 80× 9 = 720 54 6× 9 = 54 8294 1	
• Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU. • Again, the carry digits in the partial products are usually carried mentally.	286 × 29 is approximately 300 × 30 = 9000. 286 × 29 5720 286×20 2574 286× 9 8294 1	

Written methods for division

From phase 2 it shows the building up to long division through Years 4 to 6 - first long division T1s ÷ 1s, extending to HT1s ÷ 1s, then HT1s ÷ T1s, and then short division HT1s ÷ 1s.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division - for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally - for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

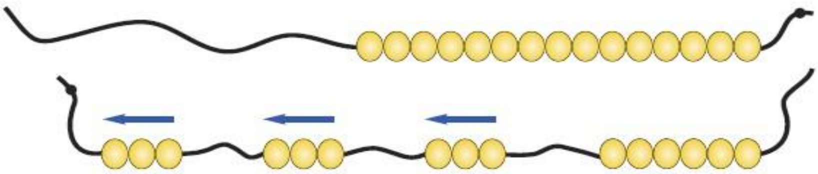
To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another - for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

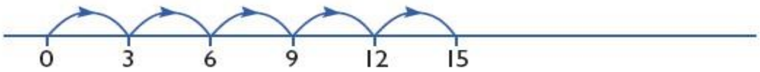
Phase 1

To half numbers to 10/20 - practically and mentally.
To be able to divide practically by sharing.

6 eggs shared between 2 nests = 3



To divide on a number line initially with no remainders and later with remainders.



$15 \div 3 = 5$

5 hops in 15. How big is each hop?

$15 \div 5 = 3$

15 shared between 5

Phase 2: Mental division using partitioning	**Phase 2**	**Key Vocabulary**
• Mental methods for dividing T1s ÷ 1s can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient. • Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention. • Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.	One way to work out TU ÷ 1s mentally is to partition T1s into a multiple of the divisor plus the remaining ones, then divide each part separately. Informal recording in Year 4 for 84 ÷ 7 might be: 84 70 + 14 ↓ ↓ ÷ 7 10 + 2 = 12 In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.	

Phase 3: Using knowledge of times tables to work out TU ÷ 1s

The apply same method for HTU ÷ 1s

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- For TU ÷ 1s there is a link to the mental method.
- As you record the division, ask: ‘How many 6s in 37?’ or ‘What is 37 divided by 6?’
- Once they understand and can apply the method, children should be able to move on from T1s ÷ 1s to HTs ÷ 1s quite quickly as the principles are the same.
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This relies on the children really knowing their times tables. Those who have not mastered them by the end of Year 4, need to be tested weekly throughout years 5 and 6.

Phase 5: Long division

Phase 3

USE PHRASE DO THE DONKEY WORK

The next step is to HT1s ÷T1s, which for children will be in Yea

The layout on the right, which links to chunking, is in essence the ‘long divi

Recording the build-up to the quotient on the left of the calculation kee with ‘chunking’ and reduces errors that tend to occu the positioning of the first digit of the quotient.

Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.

37 ÷ 6 =

Write out answers to 6 x table 6

12

18

24

30

36

6 r 1

6 | 37

Encourage children to think how many 6s made 36 and then how many more to 37.

Same works for HT1s ÷ 1s

196 ÷ 6 =

3 2 r 4

6 | 19 16

6

12

18

-

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of

Phase 5 How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples	10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. $\begin{array}{rrl} 24 & 560 & \\ & -\underline{480} & 24\times20 \\ & 80 & \\ & \underline{72} & 24\times3 \\ & 8 & \end{array}$ Answer: 23 R 8 In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below. $\begin{array}{rr} & 23 \\ 24 & 560 \\ & -\underline{480} \\ & 80 \\ & -\underline{72} \\ & 8 \end{array}$ Answer: 23 R 8	